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Fatigue Life Estimates of Mistuned Blades Via a Stochastic Approach

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This paper is an attempt to examine the vibratory characteristics of bladed disk assemblies in which the distribution of frequencies of individual blades around the rotor is random. The problem is of practical importance in view of the inevitable, though small, differences in the characteristics of blades comprising a rotor. Even if a desirable arrangement of frequencies is guided via a deterministic approach, the cost of assuring such an arrangement on the rotor of every engine manufactured can be prohibitive. Further, such an arrangement is likely to change during service because of nonuniform wear. Therefore, a preferred alternative is to develop a statistical procedure based on which the fatigue life estimates of mistuned rotors can be made. In this paper, a statistical procedure is developed and applied to obtain an estimate of expected time of failure of an assembly of 24 blades under excitation by a stationary Gaussian white noise process. The estimate is based on four blade populations consisting of 240 randomly generated frequencies—each population having the same mean but a different standard deviation. This limited study has shown that the standard deviation of the blade frequency is an important parameter that affects the average life of vibrating blades.

Nomenclature

b	= material constant
C	= material constant
C_k^a	= coefficient of aerodynamic damping
C_k^m	= coefficient of mechanical damping
$E[\cdot]$	= expected value operator
f	= frequency, Hz
f_n	= natural frequency of tuned system in Hz
$F(\omega,t)$	= input excitation force defined in Eq. (18)
$G(t)$	= stationary Gaussian random process
$h(t)$	= impulse response function
$I_k(\omega)$	= deterministic modulating function
k	= general subscript
K_k	= blade stiffness
ℓ	= general subscript
M_k	= blade mass
N	= number of blades mounted on rotor
N_s	= number of cycles to failure at stress level S
$N(\omega)$	= number of frequencies in bandwidth of excitation
$P_k(t)$	= deterministic exciting force
$P_y(s)$	= probability density function of stress amplitudes of response process $y(t)$
$q_k(t)$	= dynamic force transmitted to the disk
Q_k	= amplitude of $q_k(t)$
$R_y(t)$	= correlation function for a stationary process
$R_f(\tau)$	= correlation function of input $F(\omega,t)$
$R_G(\tau)$	= correlation function of Gaussian input
S	= stress level
S_0	= spectral density of white noise
t	= time
T	= expected time-to-fatigue failure
V	= numerical ratio defined in Fig. 3
$W_k(t)$	= disk rim displacement at position k
$\dot{W}_k(t)$	= disk rim velocity at position k
$W_k(\omega)$	= disk amplitude at position k
$y_k(t)$	= kth blade's displacement
$\dot{y}_k(t)$	= kth blade's velocity
$\ddot{y}_k(t)$	= kth blade's acceleration
$Y(t)$	= stationary random process, also blade displacement
$Z(\omega)$	= frequency dependent part of input excitation defined in Eq. (17)
$\alpha_{k\ell}$	= dynamic influence coefficients of disk
β	= Weibull distribution shape parameter
$\hat{\beta}$	= estimate of Weibull shape parameter
$\Gamma(\cdot)$	= gamma function
$\delta(\cdot)$	= Dirac delta function
ζ	= damping ratio
η	= Weibull distribution characteristic life parameter
$\hat{\eta}$	= estimate of Weibull characteristic life parameter
μ	= normal distribution mean value parameter
σ	= normal distribution standard deviation parameter
$\sigma_{yk}^2(\omega)$	= mean-square displacement response of kth blade at frequency ω
ϕ	= phase angle associated with deterministic exciting force
ψ	= phase angle associated with dynamic force
ω	= excitation frequency, rad
ω_k	= kth blade's bench frequency

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Introduction

CONSIDERABLE attention has been paid recently to the investigation of vibration problems in mistuned bladed disk assemblies.[1-6] This interest stems from the need to understand the influence of scatter in blade frequencies on the vibratory characteristics of the assembly. All rotor assemblies are essentially mistuned because there is bound to be some scatter in blade frequencies—extremely close manufacturing tolerance notwithstanding. Even a rotor with essentially no scatter in its blade frequencies develops some scatter due to nonuniform normal wear during service. Therefore, it is only the degree of mistuning that differs from one rotor to another.

The vulnerability to failure of mistuned blades depends upon a host of parameters such as 1) extent of scatter, 2) blade location around the periphery of the rotor, 3) strength of mechanical and/or aerodynamic coupling, 4) damping, and 5) input excitation intensity. Actual engine experience has indicated that blades whose frequencies lie on the low end of the spectrum are not necessarily failure prone.[6] Available analyses, which can be used to predict the influence of mistuning on forced vibration characteristics of an assembly, require an a priori knowledge of the arrangement of blades around a rotor. The number of permutations necessary to arrive at an "acceptable" arrangement of blades around a rotor can be large. Even if a desirable arrangement of blades on a problem rotor is found through an analytical study, the cost of assuring such an arrangement of blades on the rotor in every engine manufactured is obviously prohibitive. A logical alternative is to develop a statistical procedure by which the probability of failure of mistuned rotors may be determined, thereby simplifying the design analysis of bladed disk assemblies. This paper presents the details of a stochastic approach that permits an estimation of rotor fatigue life based on the characteristics of blade population and the dynamic response of the rotor under study.

The analysis outlined in this paper is based on the following set of assumptions:

1) The mistuned state refers to blades that differ from one another only in their individual frequencies.

2) The distribution of frequencies can be random.

3) The excitation to which the blades are subjected is a stationary white noise Gaussian process.

4) Palmgren-Miner hypothesis applies so that fatigue data relevant to fixed amplitude response is valid.

5) Blade response can be described by a narrow-band stationary random process with zero mean.

Furthermore, the analysis employs the following statistics: 1) mean-square displacement response, 2) expected damage, and 3) expected time to failure.

Formulation

Mean-Square Displacement Response

Let $Y(t)$ be the displacement response of a linear system subjected to a stationary random excitation. A representation of the response can be expressed as

$$Y(t) = \int h(t-\tau)F(\tau)\mathrm{d}\tau = \int h(\tau)F(t-\tau)\mathrm{d}\tau \tag{1}$$

where $F(t)$ is the random input excitation and $h(t)$ is the system impulse response function. It will be assumed that the mean displacement $E[Y(t)]=0$ and the response autocorrelation function is defined as

$$R_y(\tau) = E[Y(t)\,Y(t+\tau)] \tag{2}$$

The mean-square displacement response, σ_y^2, can be obtained from the correlation function $R_y(\tau)$ when $\tau = 0$. Thus,

$$\sigma_y^2 = R_y(0) = E[Y^2(t)] \tag{3}$$

Peak Distribution in Response Time History

The importance of the cyclic response of structures in the assessment of fatigue life is reflected in the S-N curves developed from fatigue tests. Since N_S represents the number of cycles to failure at a given stress level S, the requirement is for a method of approximating the cycles to failure undergone by a structure when randomly excited at various stress levels. When the structure being analyzed can be represented by a system of linear equations and it is slightly damped, the displacement response can be represented by a narrow-band process.‡ It has been shown that the distribution of peaks and, correspondingly, the number of cycles in the time history of a narrow-band process can be reasonably approximated by a Rayleigh distribution,[7] defined by the probability density function

$$P_y(s) = \frac{S}{\sigma_y^2}\exp\left(-\frac{S^2}{2\sigma_y^2}\right) \quad 0 \le S \le \infty \tag{4}$$

where S represents the stress level of peak amplitude and σ_y^2 is the mean-square displacement.

Expected Damage

Use of the Palmgren-Miner linear cumulative damage assumption implies that displacement $Y(t)$ can be related to stress S, i.e.,

$$N_S S^b = C \tag{5}$$

where b and C are material constants determined from experimental data. To obtain the expected damage, let N_S be the number of cycles to failure at stress level S. Then, one cycle at level S would cause an incremental amount of damage equal to $(1/N_S)$. In a fixed time T for a narrow-band process, one would expect $T\cdot f_n \cdot P_y(S)\mathrm{d}S$ cycles at this stress level where $f_n = \omega_n/2\pi$ is the natural frequency of the system in cycles per second. Therefore, the average fractional damage incurred at this stress level is:

$$[T\cdot f_n \cdot P_y(S)\mathrm{d}s]/N_S \tag{6}$$

Thus, the total expected damage at all stress levels occurring in time T is:

$$E[D] = (T\cdot f_n)\int_0^\infty \frac{1}{N_S}P_y(S)\mathrm{d}S \tag{7}$$

Expressing Eq. (5) as $N_S = C/S^b$ and substituting Eq. (4) in Eq. (7) yields

$$E[D] = \frac{T\cdot f_n}{C}\int_0^\infty \frac{S^{b+1}\exp(-S^2/2\sigma_y^2)\mathrm{d}S}{\sigma_y^2}$$

$$= \frac{T\cdot f_n}{C}\left[2^{b/2}\Gamma\left(\frac{b+2}{2}\right)\right]\sigma_y^b \tag{8}$$

Hence, the expected damage is a function of the material constants b and C and the mean-square displacement response σ_y^2.

Expected Time to Failure

Carrying the analysis one step further, the Palmgren-Miner hypothesis states that failure occurs when the expected damage equals 1. By setting $E[D] = 1$ in Eq. (8) and solving for T, the expected "time to failure" can be estimated by

$$T \cong C/f_n\left[2^{b/2}\Gamma\left(\frac{b+2}{2}\right)\right]\sigma_y^2 \tag{9}$$

where T is the would-be fatigue life if the damage rate were a constant equal to $E[D]$. However, it has been established[8] that when random stress is a narrow-band process, the

‡A narrow-band process is one in which the number of peaks equals the number of upward zero crossings, and the number of zero crossings equals the number of cycles. Narrow-band processes occur when lightly damped structures respond to random excitations.

variance of total damage near the time to failure is small. Therefore, it is reasonable to expect that T is close to the expected fatigue life for a narrow-band random process, and it serves as a useful measure in making design decisions.[7] The preceding formulation demonstrates the importance of the mean-square displacement response in the estimation of fatigue life due to vibratory excitations. In the following section it will be shown that the analytical form of his response can be derived from the equations of motion of the particular system under study.

Derivation of Mean-Square Blade Displacement Response for Bladed Disk System

Deterministic Input

The model for the bladed disk assembly is the same as used in Ref. 4. Mechanical coupling between the blades is provided for through the disk. (Details can be found in Ref. 4.) The governing equations of motion are:

$$M_k \ddot{Y}_k + C_k^a \dot{Y}_k + K_k (Y_k - W_k) + C_k^m (\dot{Y}_k - \dot{W}_k) = P_k(t) \quad (10)$$

where $k = 1,\ldots, N$; N is the number of blades in the assembly; Y_k, $\dot{Y}_k$, $\ddot{Y}_k$ represent the displacement, velocity, and acceleration of the kth blade, respectively; W_k, $\dot{W}_k$ represent the displacement and velocity of the disk at blade position k; C_k^a and C_k^m are the aerodynamic and mechanical damping coefficients; and $P_k(t) = P_{ke} i(\omega t + \phi_k)$ is a representation of the excitation force.

The dynamic force transmitted from disk to blade may be represented as

$$q_k(t) = Q_k e^{i(\omega t + \psi_k)} \quad (11)$$

Then

$$W_k(t) = \sum_{\ell=1}^{N} \alpha_{k_\ell} Q_\ell e^{i(\omega t + \psi_\ell)} \quad (12)$$

and

$$\dot{W}_k(t) = i\omega \sum_{\ell=1}^{N} \alpha_{k_\ell} Q_\ell e^{i(\omega t + \psi_\ell)} \quad (13)$$

$\alpha_{k\ell}$ are the dynamic influence coefficients of the disk. They are functions of the disk geometry, material properties, boundary conditions, rotating speed, and frequency of excitation. $\alpha_{k\ell}$ denotes the amplitude at station k on the disk due to a force of a unit amplitude at station ℓ on the disk. Substituting Eqs. (12) and (13) in Eq. (10) and rearranging the terms yields

$$M_k \ddot{Y}_k + (C_k^a + C_k^m)\dot{Y}_k + K_k Y_k = P_k(t) + (K_k + i\omega)\Sigma \alpha_{k\ell} q_\ell(t)$$
$$= [P_{k\ell}{}^{i\phi_k} + (K_k + i\omega C_k^m)\Sigma \alpha_{k\ell} Q_\ell e^{i\psi_\ell}] e^{i\omega t} \quad (14)$$

Random Input

If the exciting force is a stationary Gaussian white noise process $G(t)$ with $E[G(t)] = 0$ and $R_G(\tau) = 2\pi S_0 \delta(\tau)$, where S_0 is the intensity of the Gaussian white noise input and $\delta(\tau)$ is the Dirac delta function, then the system described by Eq. (14) becomes stochastic and the total input force can be represented by $I_k(\omega)\cdot G(t)$. $I_k(\omega)$ is a deterministic frequency and position-dependent function and $G(t)$ is the input Gaussian process. Thus, Eq. (14) will be represented by stochastic differential equations of the form

$$M_k \ddot{Y}_k(t) + (C_k^a + C_k^m)\dot{Y}_k(t) + K_k Y_k(t)$$
$$= I_k(\omega)\cdot G(t) \qquad k = 1,\ldots,N$$

where

$$I_k(\omega) = [P_k e^{i\phi_k} + (K_k + i\omega C_k^m) \sum_{\ell=1}^{n} \alpha_{k_\ell} C_\ell e^{i\psi_\ell}] \quad (15)$$

Dividing by M_k and substituting

$$\omega_k^2 = K_k / M_k \quad \text{and} \quad 2\zeta\omega_k = (C_k^a + C_k^m)/M_k$$

yields

$$\ddot{Y}_k(t) + 2\zeta\omega_k \dot{Y}_k(t) + \omega_k^2 Y_k(t) = Z_k(\omega)\cdot G(t) \quad (16)$$

where

$$Z_k(\omega) = \left[\frac{P_k e^{i\phi_k}}{M_k} + \left(\omega_k^2 + i\omega\frac{C_k^2}{M_k}\right)\Sigma \alpha_{k\ell} Q_\ell e^{\phi_\ell}\right] \quad (17)$$

and

$$F_k(\omega,t) = Z_k(\omega)\cdot G(t) \quad (18)$$

Thus, Eq. (16) can be expressed simply as

$$\ddot{Y}_k(t) + 2\zeta\omega_k \dot{Y}_k(t) + \omega_k^2 Y_k(t) = F_k(\omega,t) \quad (19)$$

Substituting Eq. (2) in Eq. (1), the correlation function for the kth blade is:

$$R_{y_k}(\tau) = E[Y_k(t)\, Y_k(t+\tau)]$$
$$= \int_{-\infty}^{\infty}\int R_{F_k}(\tau + t_1 - t_2) h(t_1) h(t_2)\, dt_1 dt_2 \quad (20)$$

The impulse response function for Eq. (16) is:

$$h_k(t) = \begin{cases} 0, & t<0 \\ \dfrac{\exp(-\zeta\omega_k t)}{\omega_k\sqrt{1-\zeta^2}} \sin\sqrt{1-\zeta^2}\,\omega_k t & t \geq 0 \end{cases} \quad (21)$$

The input correlation function is:

$$R_{y_k}(\tau) = [Z_k(\omega)]^2 R_G(\tau) = [Z_k(\omega)]^2 \cdot \frac{\pi S_0}{\zeta_{\omega k^3}}\delta(\tau) \quad (22)$$

Substituting Eq. (22) and Eq. (21) into Eq. (20), and noting that the integration is with respect to t_1 and t_2, $Z_k(\omega)$ can be removed from under the integral. Thus, for $\tau > 0$

$$R_{y_k}(\tau) = [Z_k(\omega)]^2 \cdot \frac{S_0}{\zeta\omega_{k^3}} \int_0^\infty\int_0^\infty \delta(\tau + t_1 - t_2)$$
$$\times \frac{\exp(-\zeta\omega_k t_1)}{\omega_k\sqrt{1-\zeta^2}} \sin\sqrt{1-\zeta^2}\,\omega_k t_2\, dt_1 dt_2 \quad (23)$$

When $\tau > 0$, the delta function vanishes everywhere except at $t_2 = \tau + t_1$. Integrating with respect to t_2, we obtain, after some algebraic manipulation,

$$R_{y_k}(\tau) = \frac{\pi S_0}{2\zeta\omega_{k^3}}[Z_k(\omega)]^2 \exp(-\zeta\omega_k\tau)\left\{\cos\sqrt{1-\zeta^2}\,\omega_k\tau\right.$$
$$\left. + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\sqrt{1-\zeta^2}\,\omega_k\tau\right\} \quad (24)$$

Designating Eq. (24) by $R_{y_k}(0,\omega)$ when $\tau=0$, the mean-square displacement response for the kth blade (at frequency ω) is:

$$R_{y_k}(0,\omega)=\frac{\pi S_0}{2\zeta\omega_k{}^3}[Z(\omega)]^2=\frac{\pi S_0}{2\zeta\omega_k{}^3}\left\{\left(\frac{1}{M_k}+\omega_k^2 W_k(\omega)\right)^2+\left(\omega\frac{C_k^2}{M_k}\cdot W_k(\omega)\right)^2\right\}=\sigma_{yk}^2(\omega) \quad (25)$$

Summing (or integrating) Eq. (25) over all frequencies yields

$$R_{y_k}(0)=\sum_{\omega}R_{y_k}(0,\omega)=\sum_{\omega}\sigma_{y_k}^2(\omega)=\sigma_{y_k}^2 \quad (26)$$

Thus, σ_{y_k} is the mean-square displacement for the kth blade, and is used in Eq. (9) to obtain the expected time to failure of the kth blade.

Note that $W_k(\omega)$ is the disk amplitude W_k that is calculated for all frequencies $(\omega/2\pi)$ that are part of the excitation bandwidth. By definition,

$$W_k(\omega)=\sum_{\ell=1}^{N}\alpha_{k_\ell}Q_\ell e^{i\psi_\ell} \quad (27)$$

Numerical Results

All of the numerical results presented herein pertain to the 24 bladed disk assemblies of Ref. 3. Some of the physical characteristics are:

Material of blade and disk	Stainless steel
Elastic modulus	206.85 GPa
Material density	7817 kg/m^3
Poisson's ratio	0.287

The disk is of variable thickness and has 24 blades mounted on it. The blade-alone frequency corresponding to a "tuned system" was found to be 391.8 Hz. Based on the first bending mode of vibration, a modal weight was calculated to be .008686 kg.

It should be further noted that the numerical results are confined to a 4 nodal diameter type of vibration at 0 rpm. Two values of damping ratios ($\zeta=0.001$ and 0.002) have been used.

In evaluating blade life, the constants b and C needed in the relationship $NS^b=C$ have been assumed to be 9.4 and 4.9×10^{50} for stainless steel 347. For reference purposes, expected time to failure for the "tuned system" has been computed to be 48 h ($\zeta=0.001$) and 52,036 h ($\zeta=0.002$).

Using the blade disk system with the properties just described, and a blade frequency population assumed to be normal $N(\mu,\sigma)$, where μ is the mean and σ is the standard deviation, the life distribution of rotor systems from four different blade frequency populations was estimated. The difference between the four blade populations is reflected in the standard deviation $\sigma=2$, 5, 10, and 15 Hz. The mean $\mu=391.8$ Hz is common to the four populations.

To estimate the life distribution of rotors whose blade frequencies come from a given population, a random assignment of 240 blade frequencies, obtained from the specified population§ (Ref. 9) was made. This random assignment represents the blade frequencies for ten possible arrangements of the assembly. For each rotor thus specified, the system of equations [10] was solved by a computer program

§The random blade frequencies were obtained from tables of normal random variates $N(0,1)$. The original set of 240 numbers was used to generate the blade frequencies by multiplying each number by the appropriate σ (i.e., 2, 5, 10, and 15) and then adding the mean of the 391.8 Hz to the number obtained.

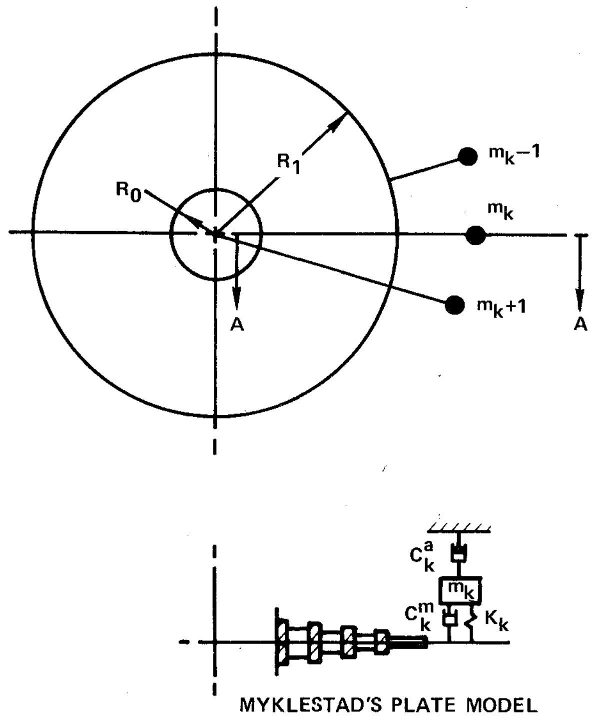

Fig. 1 **Bladed disk model.**

for a frequency bandwidth of 375-390 Hz at intervals of 1 Hz. This range of frequencies was chosen because it spans the tuned system resonance (for 4 nodal diameters and 0 rpm) of 381.5 Hz. The disk displacement, obtained by the Myklestad approach, at each blade station for each frequency was then used to calculate the blade mean-square displacement response given in Eq. (25). The average $\sigma_{y_k}^2$ for each blade (obtained by adding $\sigma_{y_k}^2(\omega)$ for each $\omega/2\pi$ and dividing by the number of frequencies in the bandwidth) was then obtained and used directly for the calculation of the expected damage [Eq. (8)] and the expected time to failure [Eq. (9)]. The shortest expected time to failure (among the 24 blades) was then noted as the time to failure for the assembly (TTFA). This procedure was repeated until ten such TTFA's were obtained; that is, ten sample points were available for the estimation of the life distribution for rotors whose blades came from a specified frequency population. The TTFA appropriate to the other three populations was obtained in a similar fashion.

Figure 1 shows the schematic diagram of the physical model. Figure 2 shows the frequency histograms of the 240 blade frequencies from populations 1-4. All four histograms have the same interval width for grouping the data and the influence of the standard deviation is reflected in the increased spread.

From each of the four populations, ten individual distributions of frequencies were assigned. The time to failure for each "rotor" was then calculated. Figure 3 highlights the effect of change in the standard deviation of the blade-frequency population upon the rotor life. Using the average life $\bar{t}$ of the set of 10 rotors, a normalized variable V is defined as the average time to failure of the population (based on 10 samples), divided by the expected time to failure of the tuned system. The natural logorithm of V is plotted against the coefficient of dispersion. (Coefficient of dispersion (CD) is the standard deviation divided by the mean $\times 100$, i.e., CD $=(\sigma/\mu)\times100$.) Since only the standard deviation has been varied from population to population, the consistent increase in the average life is directly attributed to this parameter. Figure 3 clearly illustrates the sensitivity of the standard deviation in the calculations of life of the rotor.

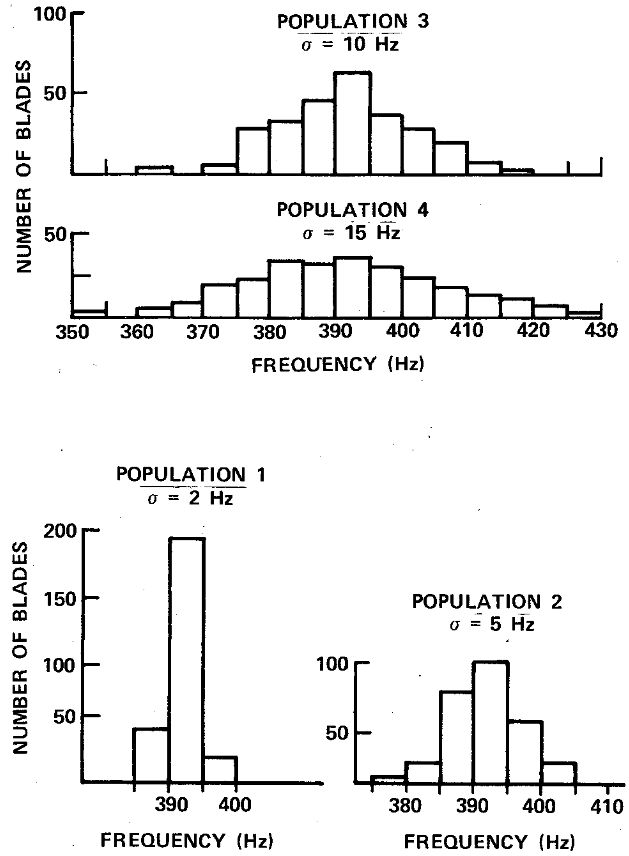

Fig. 2 Histogram of blade frequencies.

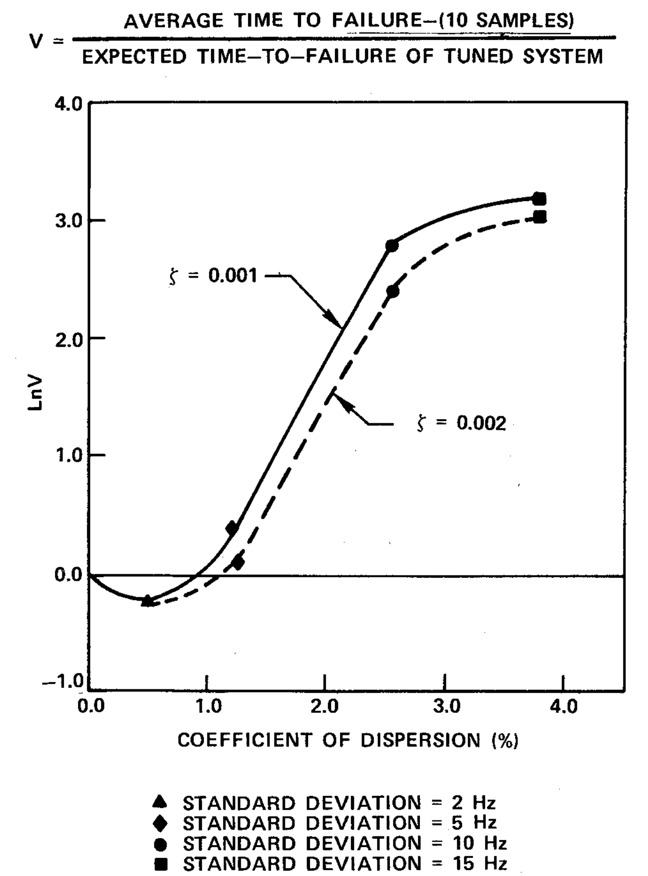

Fig. 3 Normalized "average time to failure" vs coefficient of dispersion of blade frequency population.

DAMPING RATIO = 0.001	WEIBULL ESTIMATES	
BLADE FREQUENCY POPULATION	$\hat{\eta}$	$\hat{\beta}$
▲ N (391.8 Hz, 2 Hz)	35.5	1.63
◆ N (391.8 Hz, 5 Hz)	130.0	1.08
● N (391.8 Hz, 10 Hz)	13900	0.44
■ N (391.8 Hz, 15 Hz)	72819	0.47

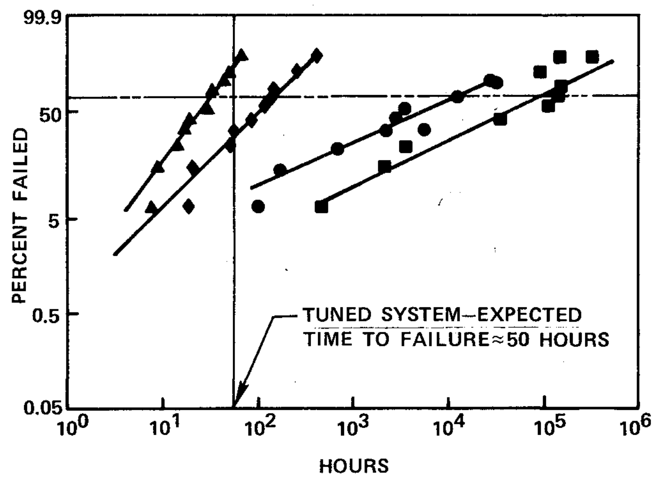

Fig. 4 Weibull plots of expected time to failure for mistuned bladed disk systems.

Perhaps the spread in the blade frequencies (as a result of the larger standard deviation of blade frequencies) enables the blades to act as tuned absorbers for each other. Whereas, a single high resonance occurs for a tuned system, a sufficiently mistuned system has multiple resonances that tend to relieve the overall stress level experienced by any blade.

The estimated life distributions are shown in Figs. 4 and 5. Figure 4 is a Weibull plot of the life distribution estimated from the ten failure times recorded for each of the four blade frequency populations when $\zeta = 0.001$. Figure 5 shows plots of the same system when $\zeta = 0.002$. As expected, increased damping increases life expectancy. By doubling the damping ratio, the tuned system life increased a thousandfold; while the mistuned systems show an increase in expected life of 400 times or more. This increase might be more consistent if a finer frequency interval were used in these calculations.

The least-square estimates of the characteristic life $\hat{\eta}$, and the shape parameter (or slope when plotted on Weibull paper) $\hat{\beta}$ are shown in the legend. The effect of the increased life for increased standard deviation is clearly depicted (Figs. 4 and 5) by the separation between the curves. The effect of the shape parameter is evident also. When $\sigma = 2$ or 5, $\hat{\beta}$ is greater than 1, and the range of life variation is more controlled; when $\sigma = 10$ or 15, $\hat{\beta}$ is less than 0.5, and the line is almost flat—thus indicating a large spread in individual life expectancy.

A third factor which critically affects the life expectancy of rotors is the environment in which they perform, limited here to the input excitation. Throughout this report the basic input excitation was assumed to be a Gaussian white noise process with intensity $S_0 = 1$. The time estimates obtained when $S_0 = 1$ can be used directly to generate time estimates for any other input excitation intensity. If T^* represents the time estimate of failure when $S_0 = 1$, then

$$T = \alpha \cdot T^* \tag{28}$$

represents the expected time to failure when $S_0 \neq 1$, if α is defined by

$$\alpha = 1(S_0)^{b/2} \tag{29}$$

DAMPING RATIO = 0.002	WEIBULL ESTIMATES	
BLADE FREQUENCY POPULATION	$\hat{\eta}$	$\hat{\beta}$
▲ N(391.8 Hz, 2Hz)	32974	1.95
♦ N(391.8 Hz, 5Hz)	77524	1.34
● N(391.8 Hz, 10Hz)	5673303	0.47
■ H(391.8 Hz, 15Hz)	38031005	0.49

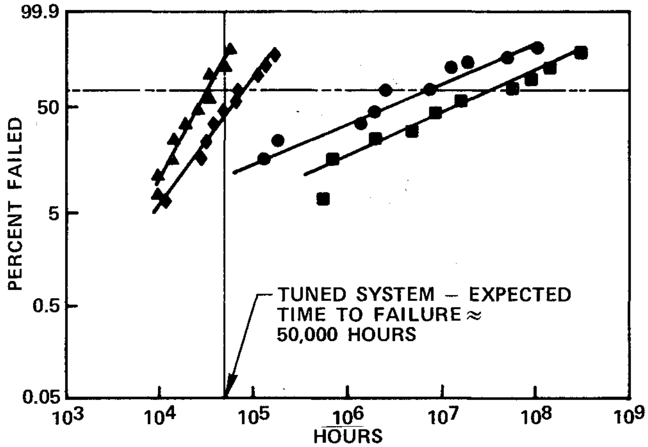

Fig. 5 Weibull plots of expected time to failure for mistuned bladed disk systems.

where S_0 is the new input intensity and b is the material constant defined for the system.

Conclusions

1) An analytic procedure for the estimation of expected fatigue life for mistuned bladed disk systems is presented. Evaluation of fatigue life by this method involves calculation of the average mean-square blade displacement over a bandwidth of frequencies in the region of the tuned system resonance.

2) Comparison of the tuned system life, when the damping ratio is increased from 0.001 to 0.002, shows the expected life increases a thousandfold. For the comparison of damping effects on mistuned systems, ten sets of randomly assigned blade frequencies from four blade frequency populations with standard deviations equal to 2, 5, 10, and 15 Hz (and the mean value equal to blade frequency of the tuned system) were analyzed. In all forty paired cases, the systems with the higher damping have a significantly longer expected life.

3) The expected life of a specific bladed disk configuration for a given intensity can be used to generate the expected life for any other input intensity by a single multiplication. A change in the input intensity does not affect the shape parameter β, but obviously does affect the characteristic life η.

4) The standard deviation of the blade frequency population is an important parameter affecting the average life of vibrating blades. When the standard deviation of the frequency population is increased, such that the coefficient of dispersion of the blade population is greater than 1.25%, the average life of the system exceeds the expected life of the tuned system. Conversely, if the coefficient of dispersion of the blade frequency population is less than 1.25%, the average life of the mistuned system is less than that of the tuned system.

5) Each blade frequency population generates its own failure distribution, which is specified by "shape" and "characteristic life" parameters β and η, respectively (Weibull distribution assumed). Since the shape parameter is represented by a line when plotted on Weibull paper, the more horizontal the line the greater the range in life expectancy. Thus, within a given blade frequency population, large differences in life expectancy can occur among the specified random patterns, especially when β is less than 1. Half of the populations analyzed in this study had a shape estimate of $\hat{\beta}$ less than 0.5. This implies a high rate of early failure.

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